Calculus Warm Up #10-4

1. $\lim\limits_{x \to \infty} 4e^{-x}\ln x$

2. Find $\dfrac{dy}{dx}$

$$x^2e^y + 3y - \ln x = 3xy + 2$$

Classwork Week 10

Warm up

Green Rev. #1

Pink Rev #2

1. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x-2}{x^2+1}$?

2. If $a \neq 0$ then $\lim_{x \to a} \frac{x-a}{x^2-a^2} =$

Answers

1) $\frac{-x^2+4x+1}{(x^2+1)^2}$

@ $x = 2 \rightarrow \boxed{\frac{1}{5}}$

$\boxed{2)\ \frac{1}{2a}}$

3. When is the graph of $y = 3x^4 - x^3 + 5x - 2$ concave down?

4. Determine the value of m that makes the following function continuous

$$f(x) = \begin{cases} mx + 2 & for \quad 0 \leq x \leq 3 \\ x^2 + 4 & for \quad 3 < x \leq 4 \end{cases}$$

3) concave down on $(0, \frac{1}{6})$

4) $m = \frac{11}{3}$

5. Given $y = 2 + xy$, find all points (x, y) on the curve where the tangent line has a slope of ½.

$$\boxed{\begin{array}{l} 5)\ (3,-1)\ \&\ (-1,1) \\ 6)\ (2x+2)e^{x^2+2x} \end{array}}$$

6. Find y' given $y = e^{x^2+2x}$

$$y - xy = 2$$
$$y(1-x) = 2$$
$$\boxed{y = \frac{2}{1-x}}$$
$$y = 2(1-x)^{-1}$$
$$y' = -2(1-x)^{-2}(-1)$$
$$y' = \frac{2}{(1-x)^2}$$

$$\frac{2}{(1-x)^2} = \frac{1}{2}$$
$$(1-x)^2 = 4$$
$$1-x = \pm 2$$
$$1 \pm 2 = x$$
$$x = 3, -1$$

7. Find the derivative of $f(x) = \ln\dfrac{\sqrt{2x-1}}{x^2(x+2)^{\frac{1}{3}}}$

$$\boxed{f'(x) = \frac{1}{2x-1} - \frac{2}{x} - \frac{1}{3(x+2)}}$$

or

$$f'(x) = \frac{-11x^2 - 11x + 12}{3x(x+2)(2x-1)}$$

8. Find the derivative of $y = (x^2)^{2x}$

$$\boxed{4x^{4x}(\ln x)}$$

$$y = x^{4x}$$

$$4(\ln x)x^{4x}$$

November 09, 2017

9. Find the following limit: $\boxed{\lim_{x\to 1^+}(x-1)^{\ln(x)}} = (1-1)^{\ln 1} = 0^0$ indeterminate

10. Find the following limit: $\lim_{x\to 1}\left(\frac{x}{x^2-1}-\frac{1}{x-1}\right) = -\infty$ (Next page)

$$\text{let } y = \lim_{x\to 1^+}(x-1)^{\ln x}$$

$$\ln y = \lim_{x\to 1^+}\left[\ln\left((x-1)^{\ln x}\right)\right]$$

$$\ln y = \lim_{x\to 1^+}\left[\ln x\left(\ln(x-1)\right)\right]$$

$$\ln y = \lim_{x\to 1^+}\left[\frac{\ln(x-1)}{\frac{1}{\ln x}}\right] = \frac{\ln(1-1)}{\frac{1}{\ln 1}} = \frac{-\infty}{\infty}$$

* Now L'Hôpital's Rule

$$\ln y = \lim_{x\to 1^+}\left[\frac{\frac{1}{x-1}}{\frac{-1}{x(\ln x)^2}}\right]$$

$$\ln y = \lim_{x\to 1^+}\left[\frac{1}{x-1}\cdot\frac{x(\ln x)^2}{-1}\right]$$

$$\ln y = \lim_{x\to 1^+}\left[\frac{-x(\ln x)^2}{x-1}\right] = \frac{-1(\ln 1)^2}{1-1} = \frac{0}{0}$$

$$\ln y = \lim_{x\to 1^+}\left[\frac{-\ln x(2+\ln x)}{1}\right] = -\ln 1(2+\ln 1) = 0$$

$$\ln y = 0$$

$$e^0 = y \longrightarrow y = 1$$

$$\boxed{\lim_{x\to 1^+}(x-1)^{\ln x} = 1}$$

9. Find the following limit: $\boxed{\lim_{x\to 1^+}(x-1)^{\ln(x)}}$

10. Find the following limit: $\lim_{x\to 1}\left(\frac{x}{x^2-1}-\frac{1}{x-1}\right) = \frac{1}{1-1}-\frac{1}{1-1}$

$$= \frac{1}{0}-\frac{1}{0}$$

$\infty - \infty$ indeterminate

$$\lim_{x\to 1}\frac{x-(x+1)}{x^2-1}$$

$$\lim_{x\to 1}\frac{-1}{x^2-1}$$

$$\frac{-1}{1-1}$$

$$-\frac{1}{0} \longrightarrow \boxed{-\infty}$$

Slope Fields (Also called direction fields)
Allow you to visualize the shape of a curve by following how the slope is changing.

A slope field is created from a differential equation.

Example: $\frac{dy}{dx} = x + 2$

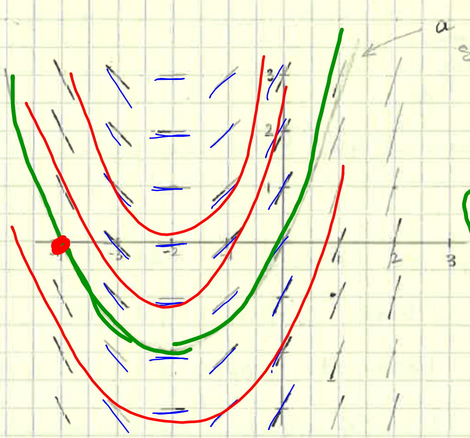
Use the differential equation to find slopes at a series of ordered pairs:

x	y	$\frac{dy}{dx}$
-2		0
-1		1
0		2
1		3
2		4

Ex: $\frac{dy}{dx} = x+2$

x	y	$\frac{dy}{dx}$
0		2
1		3
2		4
-1		1
-2		0
-3		-1
-4		-2

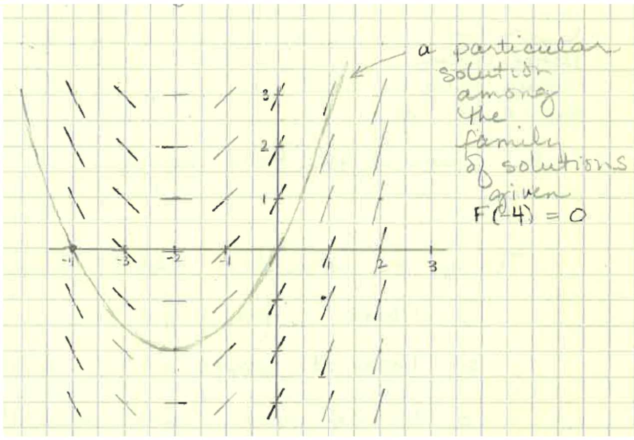
Notice when slopes only depend on x,



a particular solution among the family of solutions given $F(-4) = 0$

the columns in the slope field match (same slope at all y for that x)

A family of parabolas can be seen in the field.



The particular parabola would depend on an initial condition being given. (pt on graph)

* Notice where slope = 0 → We have a min. @ x = -2. The x-coordinate of the vertex will be -2

(Sketch particular solution when F(-4) = 0)

Things to consider about slope fields:

1) Where is the slope zero?

2) Is the slope undefined anywhere?

3) Notice when slopes depend on x only.
(The columns will match.)

4) Notice when slopes depend only on y.
(The rows will match.)

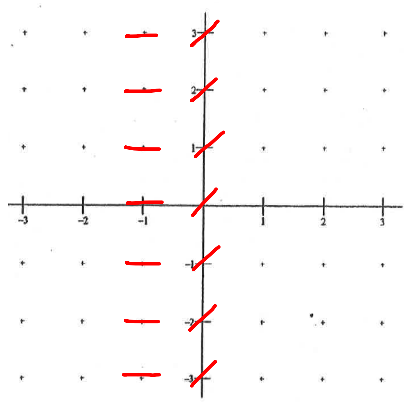
5) Notice where slopes are positive and where they are negative.

Class worksheet #1

$$\frac{dy}{dx} = x + 1$$

x	y	$\frac{dy}{dx}$
-3		-2
-2		-1
-1		0
0		1
1		2
2		3
3		4

The line segments in the slope field are called isoclines.



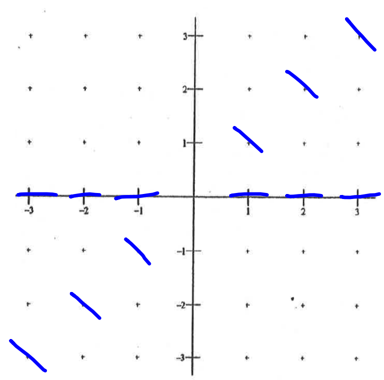
Notice that when slopes depend only on x, the columns match.

Class worksheet #6

$$\frac{dy}{dx} = -\frac{y}{x}$$

Notice slope is undefined for x = 0
Slope field will be blank along y-axis.

x	y	$\frac{dy}{dx}$
		0
		1
		-1



Sometimes the best strategy is to start with slope.

HW: Finish classwork (tan)

AP Review # 6 (yellow)

#1) $-1 + e^{1-t}$

due turned in on Tuesday

<u>Next test is Monday, Nov. 13</u>

Covers: 7.2, 7.5 & 7.8

and past topics:

evaluate limits, continuity, concavity, instantaneous rate of change, implicit differentiation